Comment on "Poynting vector, heating rate, and stored energy in structured materials: A first-principles derivation"

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In Phys. Rev. B **80**, 235120 (2009), M. G. Silveirinha criticizes our work on Poynting's theorem and energy conservation in systems with bounded media [EPL **81**, 67005 (2008)], especially with regard to our argumentation toward $\mathbf{S}=\frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B})$ as a generally valid energy flux vector. We fully rebut the criticism, point out that it is based on undue comparison, clear up some misunderstandings and show that Silveirinha's approach is rather a restricted than a general one.

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In our Letter,[1] we establish that

$$\mathbf{S}_e = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}) \tag{1}$$

is a purely electromagnetic energy flux vector (Poynting vector), which constitutes together with the electromagnetic field energy density

$$U_e = \frac{1}{2}\left(\varepsilon_0\mathbf{E}^2 + \frac{1}{\mu_0}\mathbf{B}^2\right) \tag{2}$$

and the dissipation $-\mathbf{jE}$ a universally valid energy conservation law (Poynting theorem),

$$\frac{\partial}{\partial t}U_e + \mathrm{div}\,\mathbf{S}_e = -\mathbf{jE}, \tag{3}$$

in which all quantities are well defined and have a clear physical meaning. We also point out problems that exist with the historical form of the Poynting vector,

$$\mathbf{S} = \mathbf{E}\times\mathbf{H}, \tag{4}$$

when general (realistic) media properties are considered. (In absence of magnetization, of course, both variants coincide, since $\mathbf{H}=\frac{1}{\mu_0}\mathbf{B}$ in this case.)

In Ref. 2, a "macroscopic Poynting vector" is derived together with other quantities in a composite medium, which the author eventually finds to coincide with "usual textbook formulas." Comparing his findings to ours, he blames our work for "erroneous conclusions," "unsubstantiated claims," and "fundamental misconceptions and mistakes."

We will show here that there are rather misunderstandings and mistakes from the author's part, and take the chance to clarify things from our perspective. We need to proceed carefully because the author's notion of several terms is different from ours. We underline that our results do not contradict classical textbooks,[3,4] but rather extent and concretize their accounts. The warnings contained in these books concerning the validity of Eq. (4) seemingly have been largely overlooked.

Our approach[1] starts from a consideration of the system under study as a classical ensemble of charged pointlike particles which are subjected to Newton's laws of motion and are in interaction with physical electromagnetic fields $\mathbf{E}$, $\mathbf{B}$ governed by Maxwell's equations and the Lorentz force law. This constitutes a fundamental and purely axiomatic basis for the theory. Its quantum-statistical generalization is straightforward and leads to identical relations for the expectation values. The author of Ref. 2 interprets this as a "phenomenological model," an interpretation which we strongly reject.

Matter may be arbitrarily complex, but it will always be possible to decompose it into an ensemble of charged particles. This way, there is neither a need to describe the properties of matter, nor to consider medium boundaries, to establish proper boundary conditions or even to find a proper definition of where the boundary of a medium in a microscopic picture lies.

The electromagnetic fields $\mathbf{E}$, $\mathbf{B}$ in our approach are not averaged in any sense, neither spatially nor temporally (except for the quantum-statistical expectation value being taken), and are always given in the space and time domains. Here, our notion of the term "microscopic fields" differs from that of the author, in which they are result of a "homogenization of the first level." Macroscopic fields in the sense of, e.g., Jackson[3] (spatial averaging using a phenomenological convolution function) are not employed at all in our approach.

In Ref. 2, the author imposes several important restrictions on his approach right from the start: time harmonic fields, temporal homogeneity in the system (and especially of the dielectric properties of the medium), periodicity in the material's structure, no presence of medium boundaries (i.e., infinite bulk media are assumed) and complete absence of dissipation. Medium properties are considered in an effective (i.e., homogenized) picture instead of a microscopic one. Spatial dispersion in bulk is discussed, but spatial inhomogeneity (spatial dispersion in bounded media) is not. In contrast, none of these restrictions apply to our theory of the energy flow.[1]

Consideration of spatial inhomogeneity, however, is absolutely necessary for an analytically strict description of systems with media boundaries due to the breach of translational invariance in these systems. It is impossible to describe them microscopically by a dielectric function whose spatial dependence is of the form $\varepsilon(\mathbf{r})$, as the author does, which is rather that of a spatially homogeneous system (thus with an unbounded, bulk medium). Instead, a nonlocal function $\varepsilon(\mathbf{r},\mathbf{r}')$ is needed. Obviously, our notion of the term "nonlocal function" differs from that of the author, who employs it for the Fourier-transformed form $\varepsilon(\omega,\mathbf{k})$.

In Ref. 1, we consider spatially and/or temporally inhomogeneous systems. At this level of universality, Fourier transformation of ε to wave vectors $\mathbf{k}$ or to frequencies ω is not possible. The convolution structure that arises in the relation for the displacement field,

$$\mathbf{D}(\mathbf{r},t) = \int d^3\mathbf{r}' dt' \varepsilon(\mathbf{r},\mathbf{r}',t,t')\mathbf{E}(\mathbf{r}',t'), \tag{5}$$

is a major obstacle for the description of light propagating through bounded media. Nevertheless, we were able to derive some fundamental relations and laws for such systems.[5–7]

We define the "electromagnetic Poynting vector" to be the purely electromagnetic energy flux vector $\mathbf{S}_e$, Eq. (1), that appears in the energy conservation equation, Eq. (3), which we derive from the Newton-Maxwell-Lorentz equations within a handful of algebraic manipulations only.[1] In contrast, the historical form, Eq. (4), which is also called the "Abraham" form,[8] does contain mechanical energy contributions from the movement of carriers induced in the matter (bound currents), as is pointed out in our Letter and supported by Ref. 8. The definition of "Poynting vector" in Ref. 2 is unclear in that the derivation starts from an "assumption" for the "microscopic Poynting vector."

The author claims that the "correct" form for the macroscopic Poynting vector is

$$\mathbf{S}_{av} = \frac{1}{2}\mathrm{Re}(\mathbf{E}_{av} \times \mathbf{H}_{av}), \tag{6}$$

similar to the Abraham form, and states that our Letter claims that Eq. (6) "does not apply for media with artificial magnetism." However, we did not make any statements about a Poynting vector constructed from averaged time harmonic fields in bulk media. We also note that there is no single "correct" form, as is pointed out nicely in Ref. 8. The choice is open and may depend on the system considered, the quantities of interest and the calculations to be made. However, it is to be kept in mind that each energy flux vector makes part of an energy continuity equation, whose other constituents are the energy density and a possible dissipation term. In Refs. 1 and 8, it is explicitly demonstrated how to construct different energy flux vectors and consistent energy conservation laws. Note that according to the choice of the energy flux vector, the energy represented by it and, hence, its physical meaning will also change.

The energy density belonging to the Abraham form of the energy flux vector cannot generally be given analytically, because it is mathematically not generally possible to construct an energy density whose time derivative is the corresponding term

$$\mathbf{E}\dot{\mathbf{D}} + \mathbf{H}\dot{\mathbf{B}}, \tag{7}$$

which appears in the energy continuity equation.[1] This is the case, e.g., for spatially or temporally inhomogeneous systems, see Eq. (5). Citing Landau and Lifshitz:[4] "In a dielectric medium without dispersion… this quantity [Eq. (7)] can be regarded as the rate of change of the electromagnetic energy $U=\frac{1}{2}(\varepsilon E^2+\mu H^2)$… In the presence of dispersion no such simple interpretation is possible. Moreover, in the general case of arbitrary dispersion, the electromagnetic energy cannot be rationally defined as a thermodynamic quantity." Or, citing Jackson:[3] "[The assumptions of a medium without dispersion and loss] really restrict the applicability of the simple version of Poynting's theorem to vacuum macroscopic or microscopic fields." This incompleteness is the main reason for our advice that Eq. (4) should not be considered a generally valid energy flux vector.[1] Notwithstanding, one may find assumptions and restrictions that eventually allow for the careful construction of such U even in presence of dispersion and dissipation, see examples in Refs. 4 and 3. What the author develops in Ref. 2 is one of these restricted solutions.

Under these circumstances, the approach of Ref. 2 should not be called a "completely general" "microscopic" "first-principles" approach. We do not put the author's derivation up for debate—it has to be regarded under the respective assumptions and restrictions—but we emphasize that none of our results are invalidated.

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